Coupled Dynamics of NEMS and other Driven Systems Far From Equilibrium

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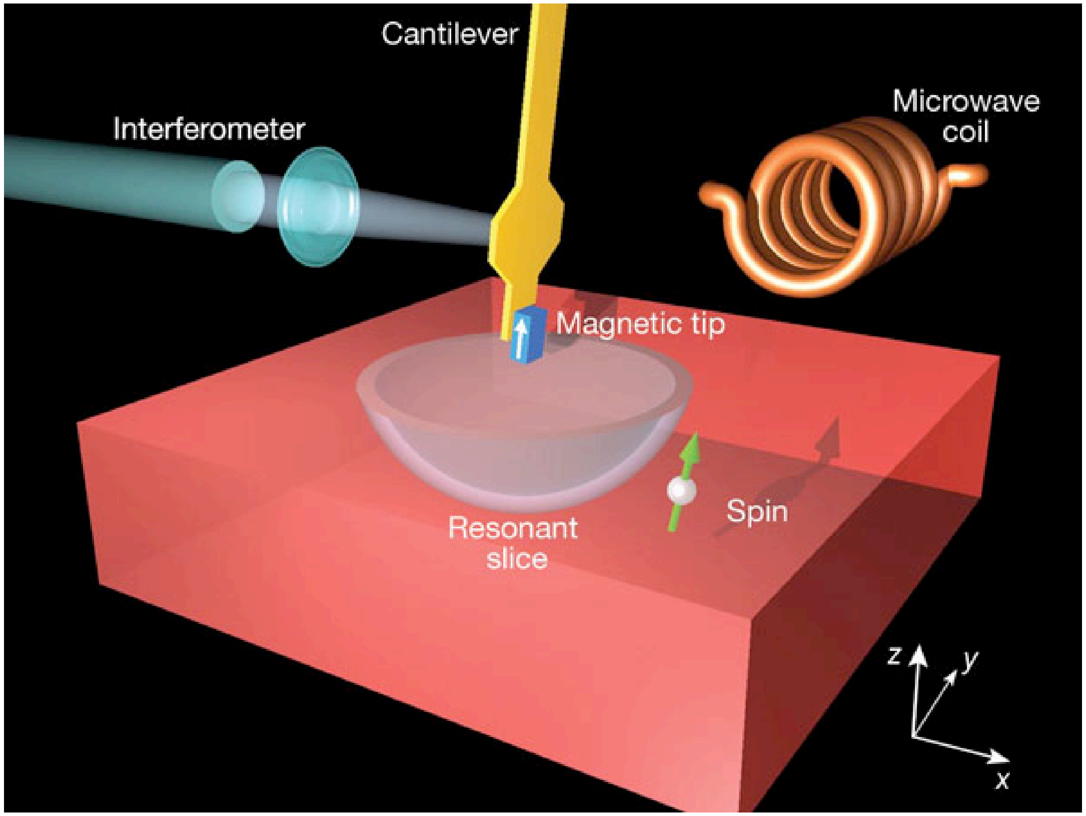
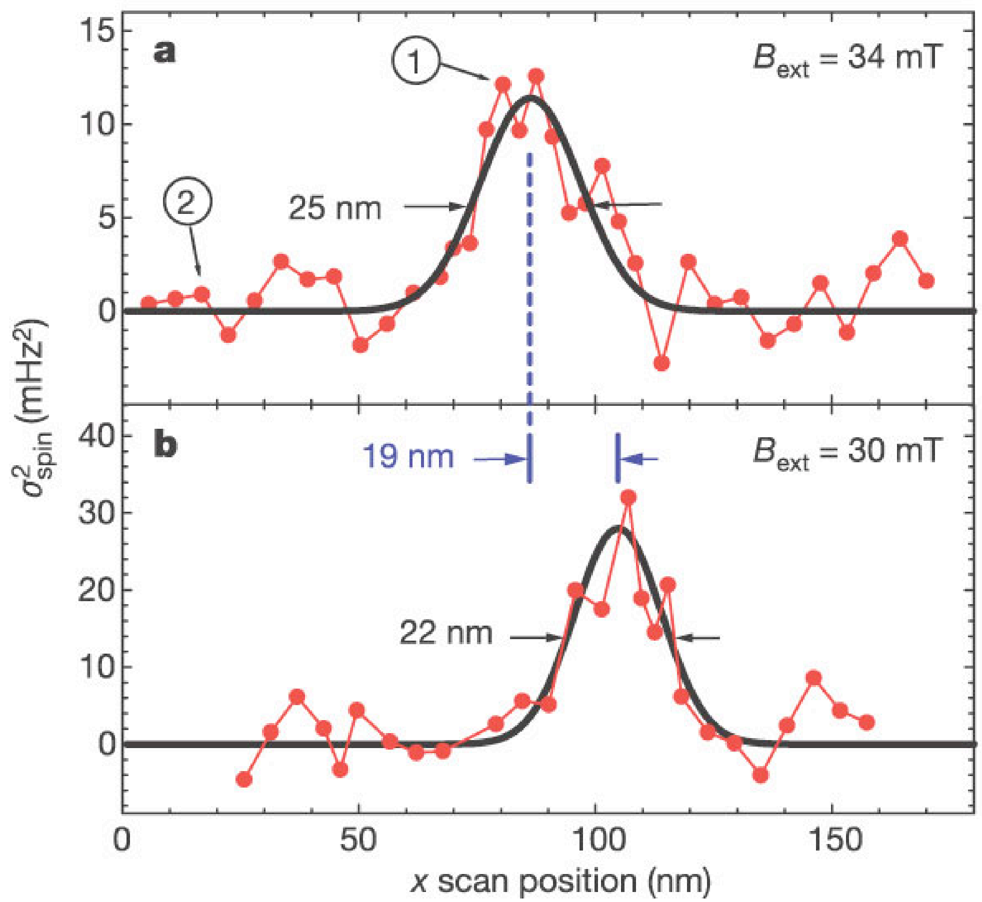
Detecting Single Spins



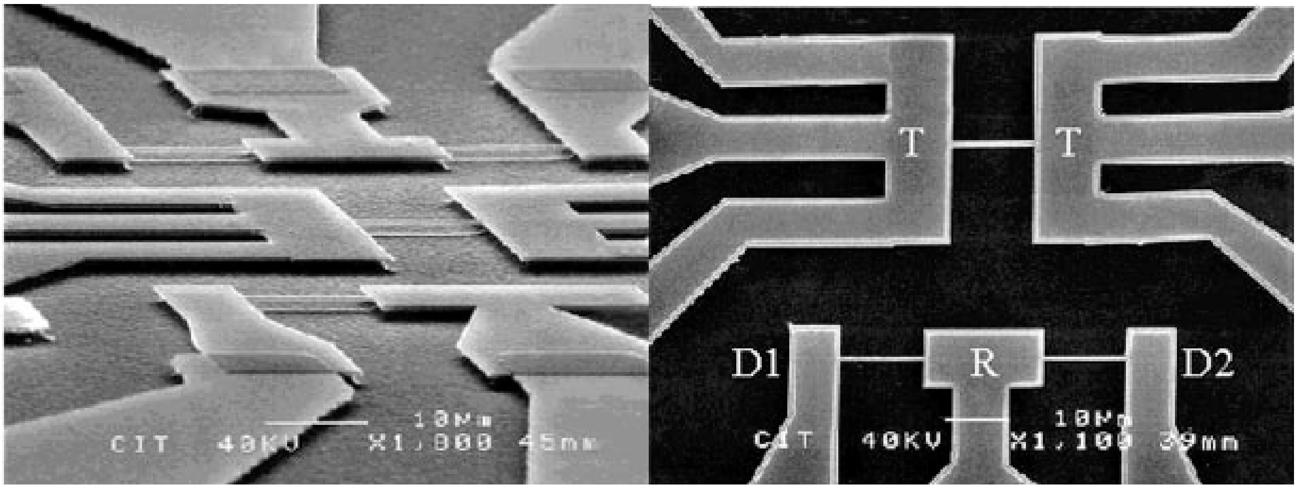
Configuration of the single-spin MRFM experiment. The magnetic tip at the end of an ultrasensitive silicon cantilever is positioned approximately 125 nm above a polished SiO_2 sample containing a low density of unpaired electron spins. The resonant slice represents those points in the sample where the field from the magnetic tip (plus an external field) matches the condition for magnetic resonance.
As the cantilever vibrates, the resonant slice swings back and forth through the sample causing cyclic adiabatic inversion of the spin. The cyclic spin inversion causes a slight shift of the cantilever frequency owing to the magnetic force exerted by the spin on the tip. Spins as deep as 100 nm below the sample surface can be probed. [D. Rugar et al., Nature **430,** 329 (2004)].

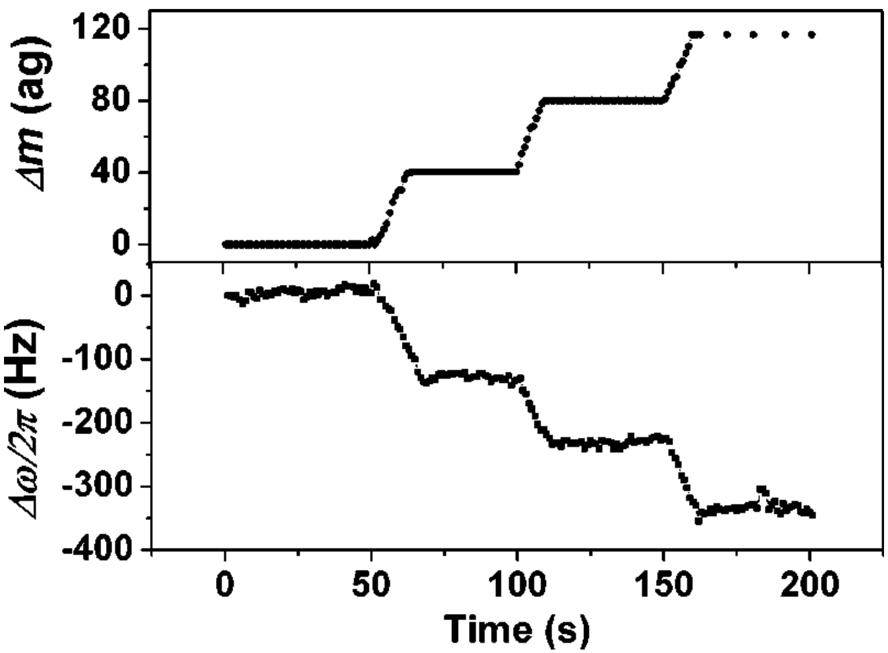


Plots showing the spin signal as the sample was scanned laterally in the x direction for two values of external magnetic field.

Ultrasensitive Mass Detection

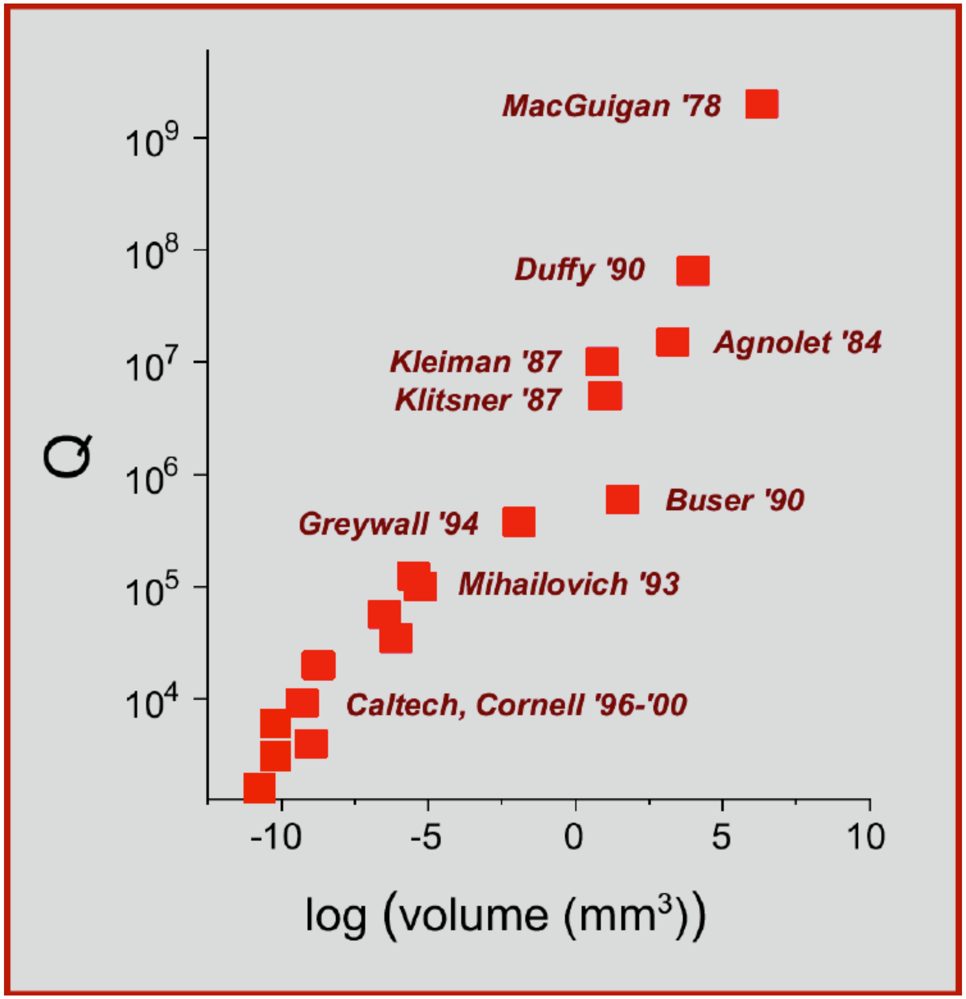
[K. Ekinci et al., Appl. Phys. Lett. **84** (2004) 4469; B. Ilic et al., J. Appl. Phys. **95** (2004) 3694]





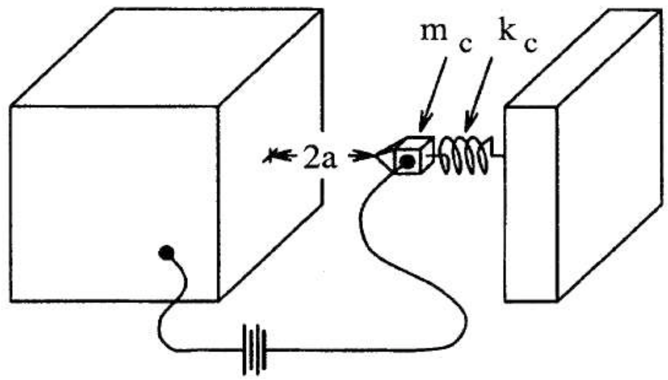
Frequency shifts (bottom) induced by sequential 40 attogram gold atom adsorption upon a doubly-clamped SiC beam resonator. Mass sensitivity is 2.5 ag for the 2.5 ms averaging time employed.

Such sensing applications require high mechanical quality factors

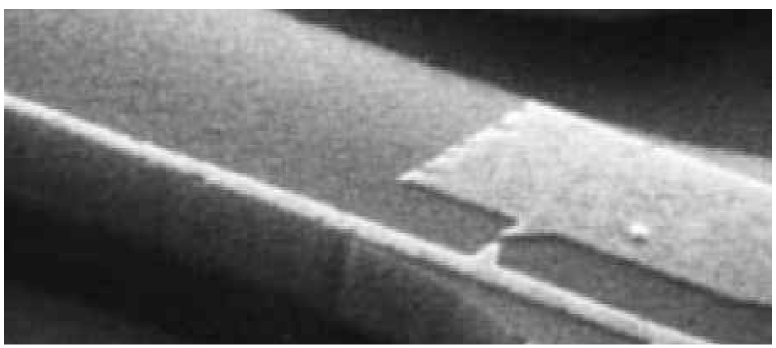


[M. Roukes, cond-mat/0008187 (after D. Harrington - unpublished)]

Examples of NEM systems:

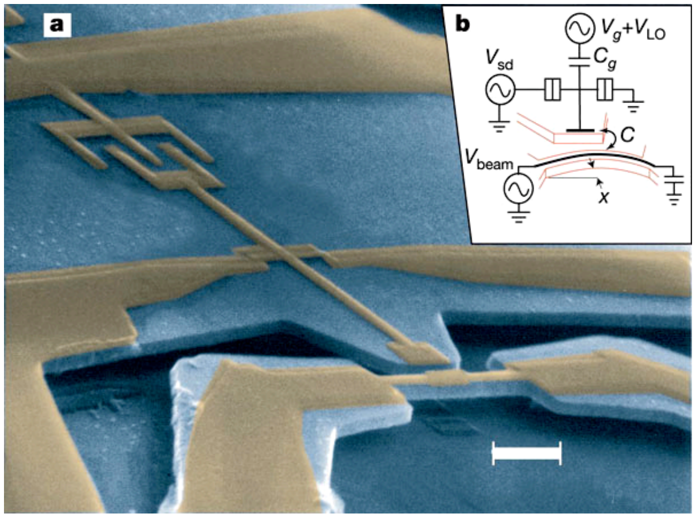


Mechanically compliant tunnel junction
[N. Schwabe et al., PRB 52, 12911 (1995)]

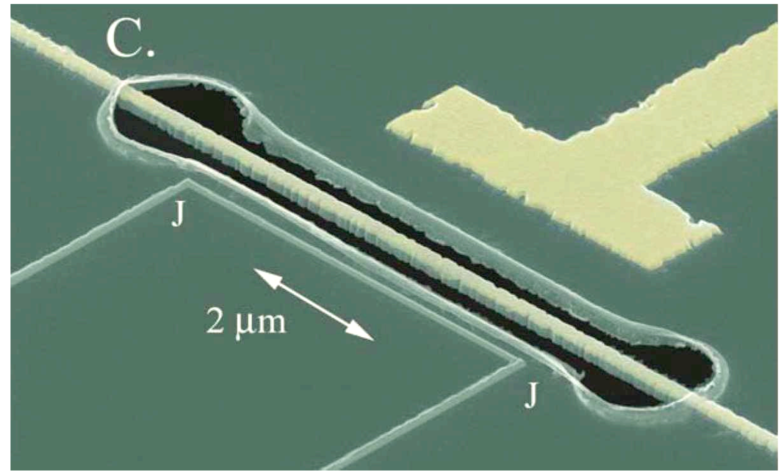


Quantum point contact displacement sensor
[A. Cleland et al., APL 81, 1699 (2002)]

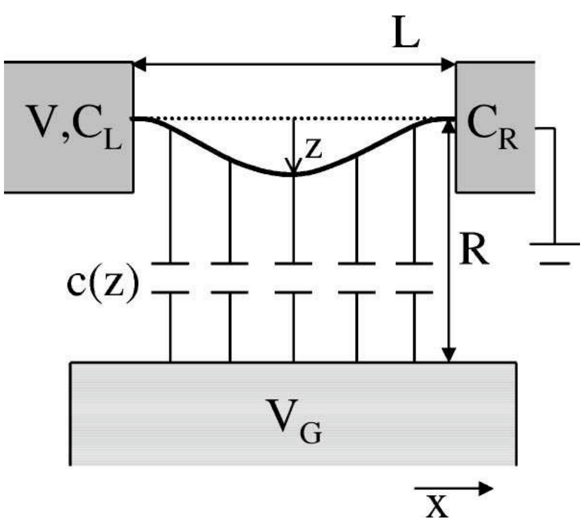
SET displacement detector:



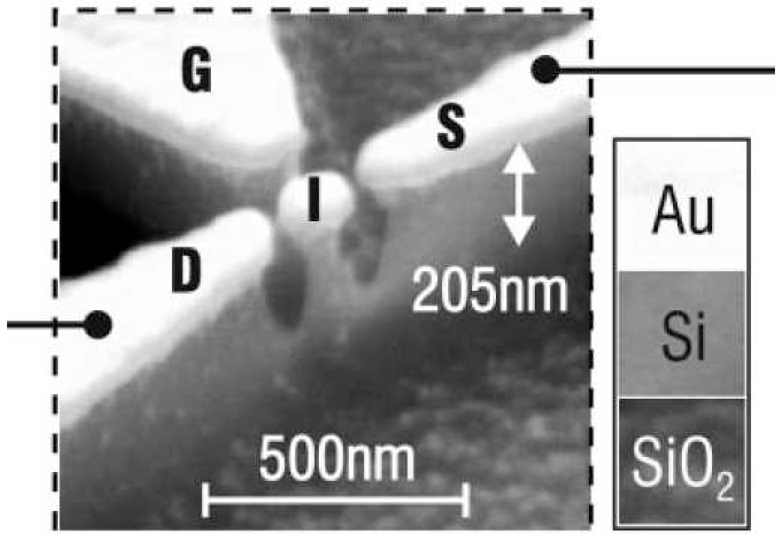
[R. Knobel et al., Nature 424, 291 (2003)]



[M. Lehaye et al., Science 304, 74 (2004)]



[S. Sapmaz et al., PRB 67, 235414 (2003)]



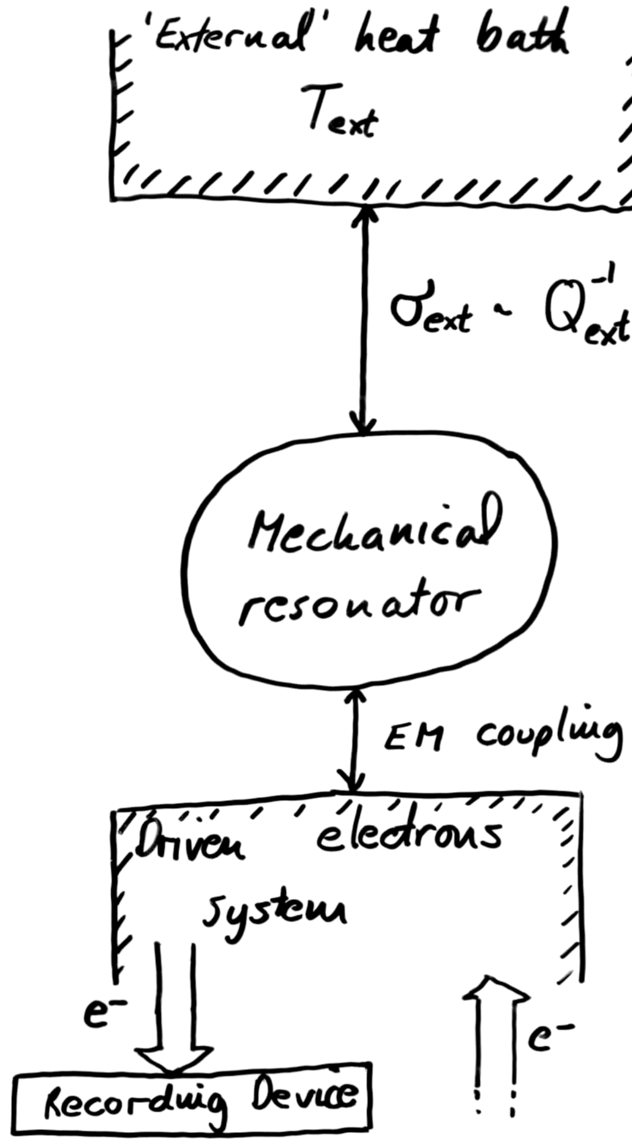
Charge shuttle
[D. Schieble et al., APL 84, 4632 (2004)]

Would like to understand the fully coupled (semi)classical and quantum dynamics of such NEMS.

Especially important to understand the back-reaction of the electronic system on the mechanical system, as measured through the former.

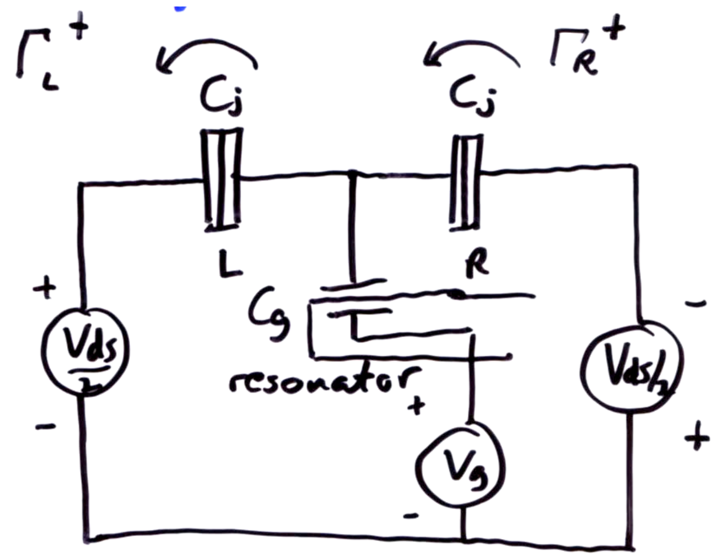
Are there common features in the coupled dynamics?

Scheme of a generic NEM system:



Classical dynamics of coupled SET-mechanical resonator system

[A. Armour et al., PRB 69, 125313 (2004)]:



Master equation for T_e=0 and *no external damping* on the mechanical resonator:

$$\dot{P}_N = \{H_N, P_N\} - (\Gamma_L^- + \Gamma_R^+)P_N + (\Gamma_L^+ + \Gamma_R^-)P_{N+1}$$

$$\dot{P}_{N+1} = \{H_{N+1}, P_{N+1}\} - (\Gamma_L^+ + \Gamma_R^-)P_{N+1} + (\Gamma_L^- + \Gamma_R^+)P_N .$$

$P_N(x,v;t)$ denotes the probability at time t that the SET island has N excess electrons and that the resonator has position x and velocity v.

Note: tunneling rates Γ are x-dependent.

Can solve analytically for the mean coordinate dynamics:

$$\langle x\rangle(t) \equiv \int dvdx\, x[P_N(t) + P_{N+1}(t)].$$

Introduce dimensionless resonator frequency and coupling strength parameters:

$\varepsilon = \omega_0\tau$, where $\tau = eR/V_{\mathrm{ds}}$ is the tunneling time, and

$\kappa = m\omega_0^2 x_0^2/(eV_{\mathrm{ds}})$, where x_0 is the difference between static positions of resonator with N and $N+1$ island electrons.

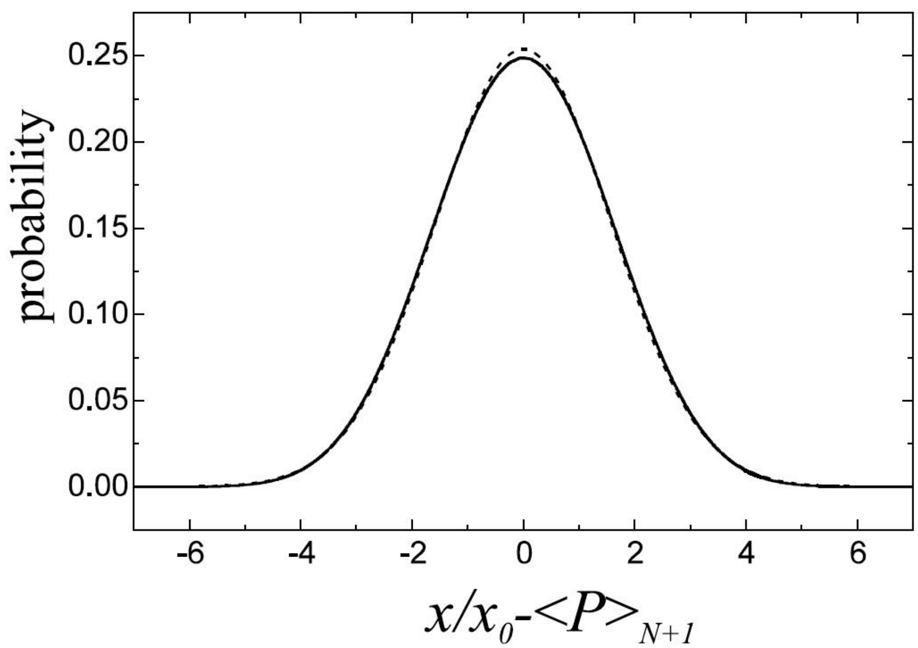
For $\kappa < 1$, and $\varepsilon << 1$, find SET *dampens* resonator with quality factor $Q = 1/(\kappa\varepsilon)$, and *renormalizes* resonator frequency: $\omega_0' = \sqrt{1-\kappa}\,\omega_0$.

Note, can reexpress this as: $\omega_0'^2 = \omega_0^2 - \gamma\Lambda$, where $\gamma = \omega_0/Q$ is the damping rate and $\Lambda = \tau^{-1}$ is the tunneling rate. C.f. oscillator system-bath (Caldeira-Leggett) model, where $\Lambda >> \omega_0$ is upper frequency cut-off in bath DOS.

Can also solve analytically for the coordinate variances. In the steady state:

$$\delta x^2 = \frac{eV_{\mathrm{ds}}}{m\omega_0^2}\langle P_N\rangle\langle P_{N+1}\rangle$$

$$\delta v^2 = \frac{eV_{\mathrm{ds}}}{m}(1-\kappa)\langle P_N\rangle\langle P_{N+1}\rangle$$



Gaussian fit to $P(x)$ using calculated position variance and mean for $\kappa = 0.1$ and $\varepsilon = 0.3$.

For $\kappa << 1$, variances satisfy <u>equipartition</u>, with $\boxed{k_{\mathrm{B}}T_{\mathrm{eff}} = eV_{\mathrm{ds}}\langle P_N\rangle\langle P_{N+1}\rangle}$

$\boxed{\equiv \text{ ensemble - averaged energy lost by tunnelling electron.}}$

Note, get identical result for tunnel-junction mechanical resonator system:

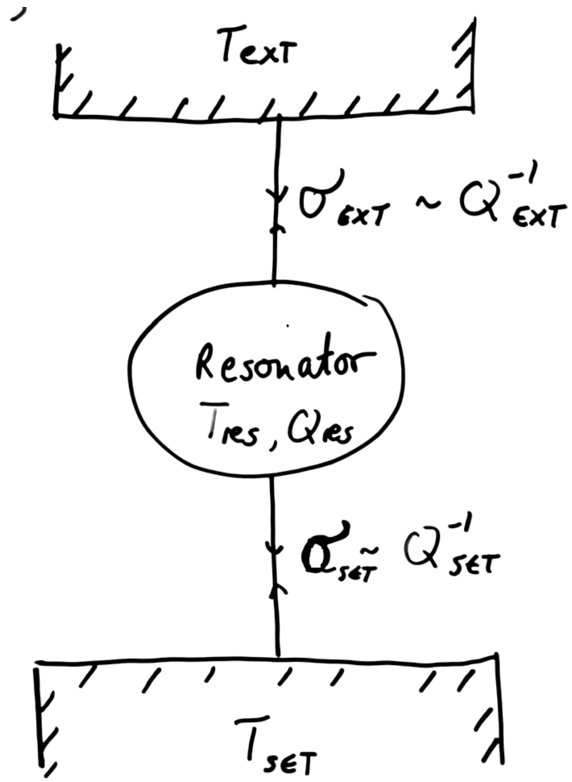
$$\boxed{k_B T_{\mathrm{eff}} = eV_{\mathrm{ds}}/2 \equiv \text{ ensemble - averaged energy lost by tunneling electron.}}$$

[D. Mozyrsky et al., PRL 89, 018301 (2002)]

Thus, in the steady state, *SET and tunnel junction behave effectively like a thermal bath for mechanical resonator probe*.

Requires weak coupling ($\kappa << 1$) and wide separation of mechanical and SET time scales.

With external (i.e., other than SET) environment, have:



$$T_{\text{res}}/Q_{\text{res}} = T_{\text{SET}}/Q_{\text{SET}} + T_{\text{ext}}/Q_{\text{ext}}$$

where $Q_{\text{res}}^{-1} = Q_{\text{SET}}^{-1} + Q_{\text{ext}}^{-1}$.

For Si cantilevers and typical SET parameters:

$$Q_{\text{SET}} = 10^{12} \frac{1}{V_g^2} \cdot \frac{t^2 d^4}{l^3 w}$$

(Cantilever dimensions in microns)

l in μm	1	5	10
f_0 in MHz	240	9.6	2.4
κ/V_g^2 in V^{-2}	3×10^{-7}	9.8×10^{-4}	0.03
$Q_i V_g^2$ in V^2	2.1×10^7	1.7×10^5	2.1×10^4

The resonant frequency, coupling strength and effective quality factor due to the SET for Si cantilevers with different lengths.

Open questions:

- Does the SET become unstable when $\kappa \to 1$?

- Does the SET also behave effectively as a heat bath when the mechanical resonator is not in the steady state?

- Under certain conditions, is there an analogous effective description for the charge shuttle and other NEMS?

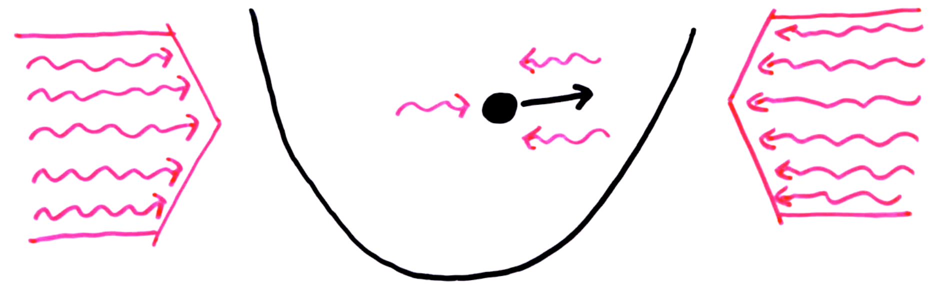
- In the steady state, how is the effective thermal motion of the mechanical resonator manifested in the SET, TJ, etc. current (noise) measurement?
[C.f. A. Armour, cond-mat/0401387; Y. Blanter et al., cond-mat/0404615; A. Clerk et al., cond-mat/0405687].

- Under what conditions does the SSET behave effectively as a heat bath? What are the analogous expressions for T_{eff} and Q_{eff} ?

- What is the analogous picture in the fully quantum description?
[C.f. A. Clerk, cond-mat/0406536 - weak coupling case].

Analogous effective thermal equilibrium behaviour observed in other far from equilibrium systems:

Laser Doppler cooling of atoms:

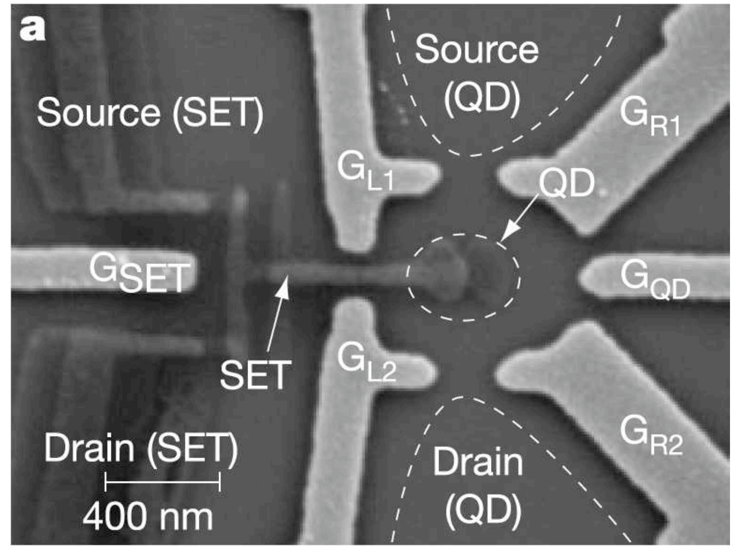


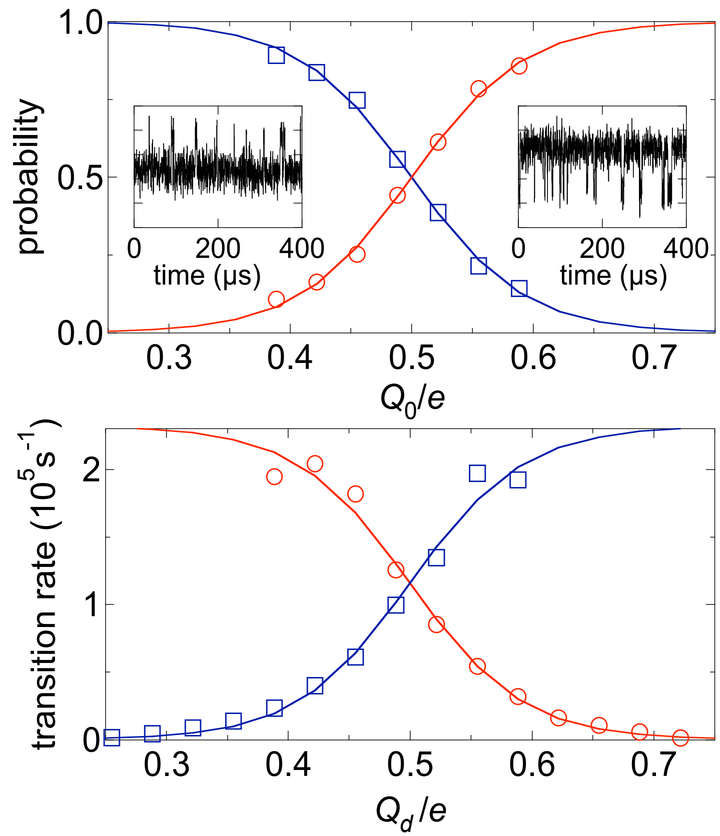
Provided the dimensionless coupling strength $\kappa = E_k^M \Big/ \frac{\hbar\Gamma}{2} << 1$, the laser beam appears to the atom as a thermal bath with minimum effective temperature $k_{\mathrm{B}} T_{\min} = \frac{\hbar\Gamma}{2}$.

E_k^M is the kinetic energy gained by the mass M atom after absorbing a photon from rest.
Γ is the decay rate of the excited level of the atom.

(C.f. $\kappa = m\omega_0^2 x_0^2 \big/ (eV_{\mathrm{ds}})$ and $k_{\mathrm{B}} T_{\mathrm{eff}} \sim eV_{\mathrm{ds}}$ for the SET)

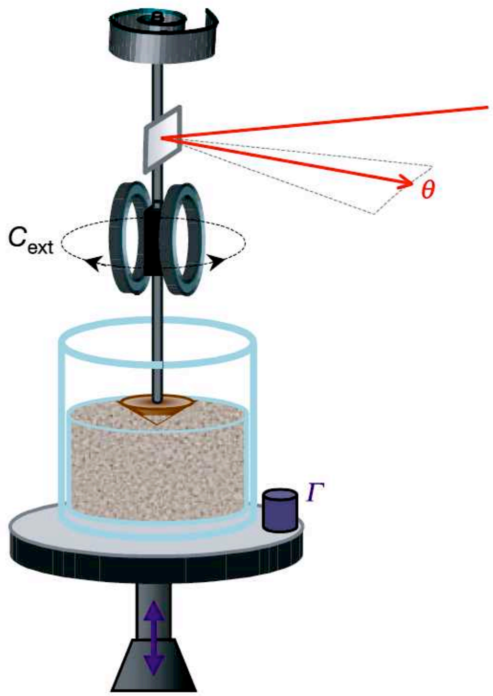
Realtime SET detection of electron tunneling in a quantum dot [W. Lu et al., Nature 423, 422 (2003)].





Effective temperature $T_{\mathrm{eff}} = 0.43$ K

Sphere mass in gas flow
[R. Ojha et al., Nature 427, 521 (2004)].



Torsional oscillator in vibrational fluidized
granular medium [G. D'Anna et al., Nature 424, 909 (2003)].